Test 3 - MTH 1410

Dr. Adam Graham-Squire, Fall 2017

16:46 ✓

Name: Key

I pledge that I have neither given nor received any unauthorized assistance on this exam.

____
(*signature*)

DIRECTIONS

1. Don't panic.
2. Show all of your work and use correct notation. A correct answer with insufficient work or incorrect notation will lose points.
3. Clearly indicate your answer by putting a box around it.
4. Cell phones and computers are not allowed on this test. Calculators are allowed on the first 4 questions of the test, however you should still show all of your work. No calculators are allowed on the last 3 questions of the test.
5. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
6. If you need it, the quadratic formula is $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
7. Make sure you sign the pledge above.
8. Number of questions = 7. Total Points = 45.

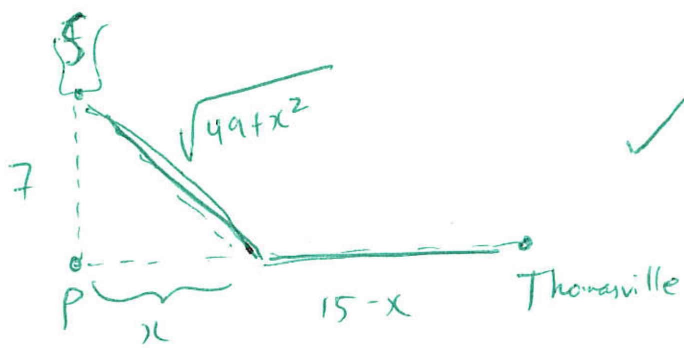
1. (8 points) Sir Topham Hatt wants to build a train track from the island of Sodor to the town of Thomasville, and he wants to do it in the most cost-effective manner. The island of Sodor lies 7 miles from the nearest point (which we will call P) on the mainland, and the town of Thomasville lies 15 miles along the straight coastline from the point P. Suppose that it costs \$200,000 per mile to build the track over the water, and \$250,000 per mile to build the track over land.

(a) Draw a diagram of the situation.

(b) Suppose the track is built directly from the island to the point P, then makes a right-angled turn and goes on land to Thomasville from there. What will be the cost of the track?

(c) Suppose the track goes directly from Sodor to some point a distance of x miles from P, then makes an obtuse-angled turn to go the rest of the way on land to Thomasville. *Use calculus* to find at what point x the track should land to make the track as cheap as possible. Round your answer to the nearest 0.01 miles.

(a)



✓

(b) Cost is $7 \cdot 250{,}000 + 15(200{,}000) = \$4{,}750{,}000$ ✓✓

(c) $\text{Cost}(x) = 250{,}000\sqrt{49+x^2} + 200{,}000(15-x)$ ✓✓

$C'(x) = 125{,}000\,(49+x^2)^{-1/2} \cdot 2x - 200{,}000$ ✓

$0 = \dfrac{250{,}000x}{\sqrt{49+x^2}} - 200{,}000$ 0.5

$200{,}000 \cdot \sqrt{49+x^2} = 250{,}000x$ 0.5

$\left(\sqrt{49+x^2}\right)^2 = \left(\frac{5}{4}x\right)^2$

$49+x^2 = \dfrac{25x^2}{16}$ 0.5

$49 = \dfrac{9}{16}x^2$ 0.5

$x = \sqrt{\dfrac{49 \cdot 16}{9}} = \dfrac{7 \cdot 4}{3} = \dfrac{28}{3} = \boxed{9.33 \text{ miles}}$

2. (6 points) Phineas and Ferb are creating a mountain in their backyard made out of little styrofoam balls. The mountain is cone-shaped, and always has the property that its base radius is 3 times the height of the cone. Assuming they are pouring styrofoam balls on the top of the cone at a rate of 400 ft^3/minute, how fast is the radius of the cone growing when the mountain is 50 feet tall? Note: the volume of a cone is $V = \frac{1}{3}\pi r^2 h$.

Round to nearest 0.001

Know $\frac{dV}{dt} = 400$

Want $\left.\frac{dr}{dt}\right|_{h=50}$

Also, $r = 3h \Rightarrow \frac{r}{3} = h$

when $h = 50$,

$r = 3 \cdot 50$

$r = 150$

$V = \frac{1}{3}\pi r^2\left(\frac{r}{3}\right)$

$V = \frac{\pi}{9} r^3$

$\frac{dV}{dt} = \frac{\pi}{9} \cdot 3r^2 \cdot \frac{dr}{dt}$

$400 = \frac{\pi}{\cancel{9}_3} \cdot \cancel{3}(150)^2 \cdot \frac{dr}{dt}$

$400 = \frac{22500\pi}{3} \frac{dr}{dt}$

$\frac{400 \cdot 3}{22500\pi} = \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = 0.01698$

$\boxed{\text{ft/sec.} \quad 0.017}$

3. (6 points) *Use calculus* to calculate the absolute maximum and absolute minimum values for

$$f(x) = \tan(x) - 8\sin(x)$$

on the interval $[0, 1.5]$.

Round to nearest 0.01

$f'(x) = \sec^2 x - 8\cos x$ ✓

$0 = \frac{1}{\cos^2 x} - 8\cos x$ ✓

$8\cos x = \frac{1}{\cos^2 x}$ ✓

$\sqrt[3]{\cos^3 x} = \sqrt[3]{\frac{1}{8}}$

$\cos x = \frac{1}{2}$

$\Rightarrow x = \frac{\pi}{3}$

✓

✓ $f(0) = 0 - 8 \cdot 0 = 0$

✓ $f(\frac{\pi}{3}) = \sqrt{3} - 8\left(\frac{\sqrt{3}}{2}\right)$

$\boxed{= -3\sqrt{3}}$

$f(1.5) = \boxed{6.12146}$

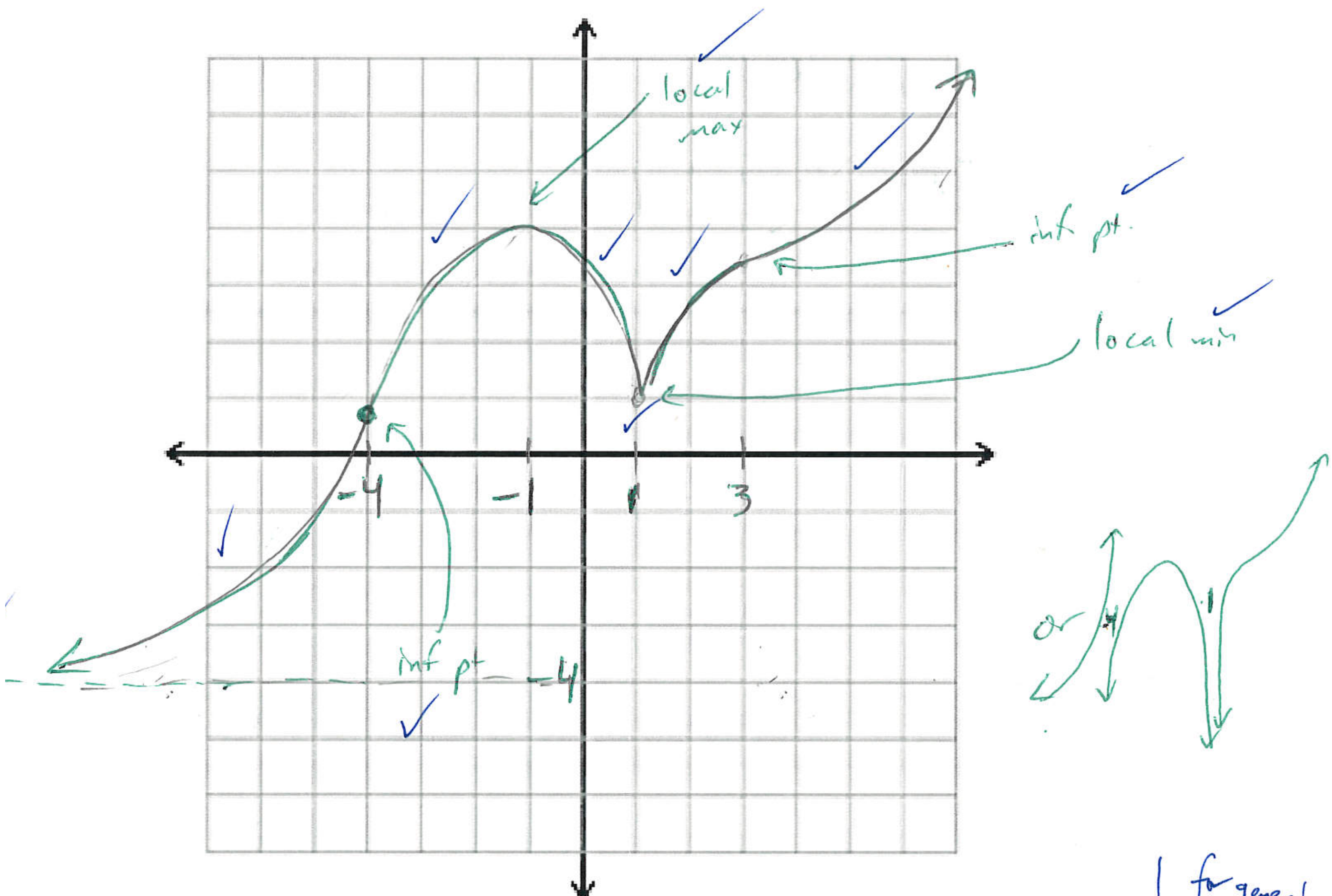
abs max $= 6.12$

abs. Min $= -3\sqrt{3}$ or -5.20

4. (6 points) Sketch a curve $f(x)$ with the following properties, and label all inflection points and local maximums and minimums (if any exist) on your graph.

- $f(1) = 1$
- $\lim_{x \to (-\infty)} f(x) = -4$
- $\lim_{x \to \infty} f(x) = \infty$
- $f'(x) > 0$ on the intervals $(-\infty, -4), (-4, -1)$, and $(1, \infty)$.
- $f'(x) < 0$ on the interval $(-1, 1)$.
- $f''(x) > 0$ on the intervals $(-\infty, -4)$ and $(3, \infty)$.
- $f''(x) < 0$ on the intervals $(-4, 1)$ and $(1, 3)$.

Note: It is possible to draw this curve with a vertical asymptote or without, either is fine.



1 for general shape

−1 if graph is good but not labeled

No Calculator Name: Key

5. (5 points) Use linearization to approximate $\sqrt{26}$.

✓ $f(x) = \sqrt{x}$ $p = 25$ ✓

$L(x) = f(p) + f'(p)(x-p)$ ✓✓

$f'(x) = \frac{1}{2\sqrt{x}}$ ✓

$L(x) = 5 + \frac{1}{10}(x - 25)$ ✓

$f(p) = \sqrt{25} = 5$ ✓

$f'(p) = \frac{1}{2 \cdot \sqrt{25}} = \frac{1}{10}$ ✓

$L(26) = 5 + \frac{1}{10}(26 - 25)$ ✓

$= 5 + 0.1 = \boxed{5.1}$ ✓

6. (8 points) (a) For the function $h(x) = x \cdot e^x$, calculate the interval(s) where h is increasing, where h is decreasing, and where h has any local maximums and/or minimums. If it has any local extrema, explain whether/which ones are maximums and which are minimums.

x-values

$$h'(x) = x \cdot e^x + e^x = (x+1)e^x \checkmark$$

$$0 = (x+1)e^x \Rightarrow x = -1 \checkmark$$

$-$ $+$

-1 $\checkmark$

$h'(-4) = -$

$h'(0) = +$

$$\boxed{\begin{array}{l}\text{decreasing on } (-\infty, -1) \\ \text{increasing on } (-1, \infty) \\ \text{local } \underline{\text{min}} \text{ at } x = -1\end{array}}$$

$\checkmark$

(b) For the function $h(x) = x \cdot e^x$, calculate the interval(s) of concavity and the x-value(s) for inflection points, if any exist.

$$h''(x) = (x+1)e^x + 1 \cdot e^x \Rightarrow (x+2)e^x \checkmark$$

$$0 = (x+2)e^x \Rightarrow x = -2 \checkmark$$

$h''(-3) = -$

$-$ $+$

-2 $\checkmark$

$h''(0) = +$

$$\boxed{\begin{array}{l}\text{Concave } \underline{\text{down}} \text{ on } (-\infty, -2) \\ \text{" up on } (-2, \infty) \\ \text{inf. pt at } x = -2\end{array}}$$

$\checkmark$

7. (6 points) Calculate the most general antiderivative of

$$f(x) = \frac{x^6 + x^2}{x^3} + \frac{1}{\sqrt{1 - x^2}}$$

$$f(x) = \frac{x^6}{x^3} + \frac{x^2}{x^3} + \frac{1}{\sqrt{1-x^2}}$$

$$f(x) = x^3 + \frac{1}{x} + \frac{1}{\sqrt{1-x^2}}$$ 0.5

$$\int f(x) = \int \left(x^3 + \frac{1}{x} + \frac{1}{\sqrt{1-x^2}} \right) dx$$

$$= \boxed{\frac{x^4}{4} + \ln|x| + \arcsin(x) + C}$$

0.5

Extra Credit(up to 2 points) You can choose either 1 point extra credit or 2 points extra credit. If you choose 1 point you are guaranteed to get the 1 point. If you choose 2 points, though, and more than 2 students (including yourself) choose 2 points, though, everyone who chooses 2 points (including yourself) gets nothing.

1 point	2 point	0 points
9	2	1